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FINAL REPORT

CONTRIBUTIONS TO THE MONO LAKE EXPERIMENTS

Research conducted for:

Department of the Navy
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VOLUME II

THE LINEAR THEORY OF WATER WAVES GENERATED BY EXPLOSIONS

Prepared by:
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FOREWORD

This is Volume II of the final report under Contract No. Nonr 5006(00). This report is issued in order to present the linear theory (in its most general form) of water waves produced by explosions and to specify the assumptions which prevail in performing the Mono Lake deep water predictions. The most general form of the linear theory has never appeared in the literature prior to this report and it is evident that it has not been employed to its fullest extent.

This report was prepared under the direction of Dr. B. Le Méhauté, Associate Director and Project Manager for this program. Miss Lois Webb contributed to the development of the computing techniques and the programming was accomplished by Mr. H. Lee Butler.

ABSTRACT

The objective of this project was to predict the water waves generated by the Mono Lake field tests. These tests consisted of detonating ten TNT explosions of approximately 9,500 pounds each, and measuring the water waves and run-up produced by these detonations.

Volume I of the final report contains these predictions and a brief description of the theoretical and empirical methods employed in performing the predictions. This volume presents the general form of the linear theory of water waves produced by explosions in water of constant depth. The theory is developed in detail so that the full extent of the assumptions made in performing the predictions for Mono Lake can be realized. It is also evident that several natural extensions to the presently employed theory can be made which will result in more realistic initial conditions and which should present a more reliable theory. Examples of the various solutions that depict the differences between a symmetric surface deformation, a symmetric time-dependent surface deformation, and an asymmetric time-dependent surface deformation are given.

CONTENTS

1. INTRODUCTION

The linear theory of water waves produced by explosions has been utilized by many investigators (Lamb 1945; Van Dorn 1959, 1963; Kranzer and Keller 1955; Prins 1956; Whalin 1965; Kajiura 1963; Unoki and Nakano 1953; Stoker 1947; Penney 1950; Fuchs 1952; Kirkwood and Seeger 1950) during the last 20 years. The basic mathematical foundation for the generalized linear theory has been in existence for a longer period of time. It involves the use of a time-dependent Green's Function which satisfies the Laplace equation with the appropriate linearized boundary conditions and the recognized assumptions that linearization involves. However, this theory in its general form has escaped application with one notable exception (Kajiura, 1963), who applied the theory to calculate the leading wave of a tsunami and gave various examples of the theory to show that certain special cases reduced to the well known solutions (Kranzer and Keller, 1955) for explosion waves.

This report formulates the general theory and applies it to water waves produced by explosions. The asymptotic solutions are developed, along with examples, for several special cases and the full impact of the use of the time-dependent Green's Function is thoroughly discussed. The advantage of using this theory in the case of one detonation and multiple detonations is pointed out.

The axially symmetric water craters presently being used to predict explosion waves many times bear little resemblance to the actual craters created (Van Dorn 1963, Whalin 1965); these craters are generally termed "pseudo" craters. The use of a time-dependent initial deformation will undoubtedly result in much more realistic crater parameters. These parameters should be related to weapon yield and location as has been accomplished for stationary symmetric water craters (Whalin, 1965). The theory in its most general form can

be applied to predict the wave spectrum generated by an exotic shaped source resulting from multiple detonations which may be employed to produce wave amplification at a certain point or in a specified area.

The conclusion of this report is that the prediction methods presently employed and used for the Mono Lake program should be modified to account for a time-dependent source of disturbance. Further, the empirical relationships between the cavity parameters and the weapon size and location should be established as they have been for a stationary initial deformation. Furthermore, one can probably establish a model, which will account for the evaporation of water as well as the subsequent return of water from the plume collapse. In addition, the general solution for an arbitrary shaped initial deformation can be used to predict the wave spectrum generated by multiple explosions.

2. THEORETICAL FORMULATION

The mathematical model employed in the generalized linear theory of water waves generated by explosions is based on the linear theory of surface waves in an inviscid incompressible fluid of constant depth. This model allows the source to be asymmetrical and time-dependent in its most general form. Consequently, a mathematical model is available in which the actual wave generation mechanism can be simulated much more precisely than that generally used in present applications.

The water is assumed incompressible, of constant depth d; the origin of the Cartesian coordinate system (x', y', z') is at the undisturbed free surface, the vertical axis z' is positive upwards, and the variables are transformed to dimensionless form (i.e., $x = x'/d$, $y = y'/d$, $t = t'\sqrt{g/d}$, $\eta = \eta'/d$, $V = V'/\sqrt{gd}$; $\varphi = \varphi'/d\sqrt{gd}$; $p = (p'/\rho)/gd$; etc.).

The kinematic and dynamic conditions at the free surface are

$$\varphi_z = \eta_t, \; z = 0 \tag{1}$$

$$\varphi_t = -\eta - p, \; z = 0 \tag{2}$$

and the bottom condition is

$$\varphi_z = w_B, \; z = -1 \tag{3}$$

where w_B is the assumed bottom velocity corresponding to a bottom deformation. The above conditions are subject to the restrictions of assuming a linear approximation. That is, the deformation at the surface and bottom are assumed small compared with the wave length, λ', and water depth, d, and the condition $z'\lambda'^2/d^3 \leq 1$.

The time-dependent Green's Function, G, is a solution of the Laplace equation

$$\nabla^2 G = 0, \quad 0 > z > -1, \quad t \geq \tau \tag{4}$$

satisfying the free surface condition

$$G_{tt} + G_z = 0, \quad z = 0 \tag{5}$$

and the bottom condition

$$G_z = 0, \quad z = -1 \tag{6}$$

It is required that G, G_x, G_y, G_z, G_t, G_{tx}, G_{ty}, and G_{tz} be uniformly bounded for every t at $x, y \rightarrow \infty$. In addition, $(G - 1/R)$ must be bounded at (x_o, y_o, z_o) where

$$R^2 = (x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2 \tag{7}$$

At time $t = \tau$ the following initial conditions are assumed

$$G = G_t = 0, \quad z = 0$$

The above equation and conditions are sufficient to determine G uniquely. From Stoker, the Green's Function for the case of three-dimensional motion in water of finite depth is

$$G(x_o, y_o, z_o; \tau | x, y, z, t) = \int_0^\infty \frac{J_o(m\bar{r})}{\cosh m} \left[\sinh m \left\{ 1 - |z - z_o| \right\} - \sinh m \left\{ 1 + (z + z_o) \right\} \right.$$

$$\left. + \frac{2}{\gamma^2} \left\{ 1 - \cosh \gamma (t - \tau) \right\} \frac{m}{\cosh m} \cosh m(1 + z) \cosh m(1 + z_o) \right] dm,$$

$$0 > z, \qquad z_o > -1$$

where

$$\bar{r}^2 = (x - x_o)^2 + (y - y_o)^2 \tag{10}$$

and

$$\gamma^2 = m \tanh m \tag{11}$$

Note that G is symmetric with respect to (x_o, y_o, z_o) and (x, y, z) and also t and τ. Therefore

$$G(x_o, y_o, z_o; \tau | x, y, z; t) \equiv G(x, y, z; t | x_o, y_o, z_o; \tau) \equiv G(x_o, y_o, z_o; t | x, y, z; \tau) \tag{12}$$

From Green's formula and the above Green's function

$$\varphi_\tau (x, y, z; \tau) = \frac{1}{4\pi} \iint_S \left(G\varphi_{\tau z_o} - \varphi_\tau G_{z_o} \right)_{z_o = 0} dS_o$$

$$- \frac{1}{4\pi} \iint_S \left(G\varphi_{\tau z_o} - \varphi_\tau G_{z_o} \right)_{z_o = -1} dS_o \tag{13}$$

where $dS_o = dx_o\, dy_o$ and the integral on the lateral boundary in water vanishes because of the condition imposed on G.

The integration of Eq. 13 with respect to τ from 0 to t and substitution of conditions in Eqs. 1, 2, 5, 6 and 8 for φ and G yield

$$\varphi(x, y, z, t) - \varphi(x, y, z, 0) = - \frac{1}{4\pi} \iint_S \left[\left(G\varphi_{z_o} - G_\tau \eta \right)_{\tau = 0} + \int_0^t pG_{\tau\tau} d\tau \right]_{z_o = 0} dS_o$$

$$- \frac{1}{4\pi} \iint_S \int_0^t \left(G\varphi_{z_o \tau} \right)_{z_o = -1} d\tau\, dS_o \tag{14}$$

The first integral in Eq. 14 represents the contribution of the surface condition and the latter integral, that of the bottom condition.

At the surface $z = 0$, Eq. 9 becomes

$$G(x_o, y_o, z_o; \tau | x, y, 0, t) = \int_0^\infty \frac{2}{\gamma^2} \{1 - \cos\gamma(t-\tau)\} m \cosh m(1+z_o) \frac{J_o(m\bar{r})}{\cosh m} \, dm \tag{15}$$

and since the surface elevation, η, is given by Eq. 2, the substitution of Eq. 14 into Eq. 2 yields

$$(\eta + p) = \frac{1}{4\pi} \iint_S (I_1 + I_2 + I_3) \, dS \tag{16}$$

where

$$I_1 = \left(G_t \varphi_{z_o} - G_{\tau t}\, \eta\right)_{\tau=0}; \; z_o = 0, \; z = 0, \tag{17}$$

$$I_2 = \int_0^t p\, G_{\tau\tau t}\, d\tau + \left(p G_{\tau\tau}\right)_{\tau=t}, \quad \text{or} \tag{18}$$

$$I_2 = \int_0^t p_\tau\, G_{\tau t}\, d\tau - \left(p G_{\tau t}\right)_{\tau=0}; \; z_o = 0, \; z = 0, \text{ and} \tag{19}$$

$$I_3 = \int_0^t G_t\, \varphi_{z_o \tau}\, d\tau, \quad \text{or} \tag{20}$$

$$I_3 = -\int_0^t G_{t\tau}\, \varphi_{z_o}\, d\tau - \left(G_t\, \varphi_{z_o}\right)_{\tau=0}; \; z_o = -1, \; z = 0. \tag{21}$$

Note that Eq. 17 represents the contribution of the initial velocity and elevation of the water at the surface, Eq. 18 or Eq. 19 the contribution of the surface pressure, and Eq. 20 or Eq. 21 the contribution from the bottom deformation.

3. ASYMPTOTIC SOLUTIONS

This section presents the asymptotic solutions obtained by an application of the method of stationary phase to evaluate the integral in Eq. 16. The special cases for which the asymptotic solution is developed are a symmetric initial impulse, a symmetric initial deformation, a symmetric time-dependent initial deformation and an asymmetric time-dependent initial deformation. In each case the source is given as an arbitrary function where special cases are easily investigated. Examples of a special case appear for each assumed initial condition. The condition of a symmetric initial impulse (Section 3.1) and a symmetric initial deformation (Section 3.2) were solved by Kranzer and Keller (1955) and are reproduced here for completeness. However, the developments of Sections 3.3 and 3.4 are original and bear further investigation as far as application to actually performing predictions are concerned.

3.1 Axially Symmetric Initial Impulse

The special case of an axially symmetric initial impulse was solved by Kranzer and Keller by the method of integral transforms. When the proper assumptions are employed their solution is obtained from Eq. 16. Cylindrical coordinates are used and the initial conditions become:

a) The water surface is initially undisturbed

$$\eta_o(r_o, \theta_o, \tau) = 0$$

b) The initial velocity at the surface at time t = 0 is.

$$\varphi_{z_o}\Big|_{\substack{z_o = 0 \\ z = 0 \\ \tau = t = 0}} = 0$$

Consequently $I_1 = \left(G_t\varphi_{z_o} - G_{\tau t}\eta\right)_{\tau=0;z_o=0,z=0} = 0$

c) The applied pressure at the surface is impulsive at $\tau = 0^+$, i.e., $p = I_o\,\delta(\tau)$; therefore

$$I_2 = \int_0^t p\,G_{\tau\tau t}\,d\tau + \left(p\,G_{\tau\tau}\right)_{\tau=t},\quad z_o = 0,\ z = 0$$

$$= I_o(r_o)\,G_{\tau\tau t},\quad \tau = 0,\ z = 0,\ z_o = 0$$

d) $I_3 = 0$

Upon substitution in Eq. 16 and expressing the wave amplitude in cylindrical coordinates

$$\eta(r,\theta,t) = \frac{-1}{4\pi}\int_0^\infty 2\gamma\, m \sin\gamma t \left\{\int_0^{2\pi}\int_0^\infty I_o(r_o)\,J_o(m\bar{r})\,r_o dr_o d\vartheta_o\right\} dm \qquad (22)$$

or upon integration with respect to r_o and θ_o the wave amplitude becomes

$$\eta(r,\theta,t) = -\int_0^\infty m\sqrt{m\tanh m}\;\overline{I_o}(m) J_o(mr)\sin\left(\sqrt{m\tanh m}\;t\right)dm \qquad (23)$$

where

$$\overline{I_o}(m) = \int_0^\infty I_o(r_o)\,J_o(mr_o)\,r_o\,dr_o \qquad (24)$$

The above integral is evaluated by the method of stationary phase upon the assumption that the frequency of oscillation of $J_o(mr)$ and the circular function is large compared to that of $\overline{I_o}(m)$. Then $J_o(mr)$ is approximated by an asymptotic expansion and upon integration

$$\eta(r,t) = \frac{\overline{I_o}(m)m}{r}\sqrt{\frac{\varphi(m)\tanh m}{-\varphi'(m)}}\ \sin\left(mr - \sqrt{m\tanh m}\;t\right) \qquad (25)$$

where

$$\varphi(m) = \frac{1}{2}\sqrt{\frac{\tanh m}{m}} + \frac{1}{2\cosh^2 m}\sqrt{\frac{m}{\tanh m}} = \frac{r}{t} \qquad (26)$$

Table I in Section 3.2 gives $\overline{I_o}(m)$ for various initial impulses $I_o(r_o)$. Figure 1 shows an example of the wave amplitude as a function of time for the initial impulse in example 2 of Table I at one observation station.

3.2 Axially Symmetric Surface Deformation

The asymptotic solution for an axially symmetric surface deformation was solved by Kranzer and Keller for several special cases. However, the theory is presented in the following pages from the more general mathematical formulation given in Section 2. When cylindrical coordinates are used, the initial conditions become:

a) The water surface is initially undisturbed and is defined by the function $\eta_o(r_o)$.

b) The initial velocity of the surface at time t = 0 is

$$\varphi_{z_o}\Big|_{\substack{z_o = 0 \\ z = 0 \\ \tau = t = 0}} = 0$$

c)

$$I_1 = - G_{\tau t}\, \eta_o(r_o)\big|_{\tau=0,\, z_o=0,\, z=0}$$

$$= \left[- \int_0^\infty 2 \cos (\gamma t)\, m\, J_o(m\bar{r})\, dm\right] \eta_o(r_o)$$

d) $I_2 = I_3 = 0$

Upon substitution in Eq. 16 and expressing the wave amplitude in cylindrical coordinates

$$\eta(r,t) = \frac{-1}{4\pi} \int_0^\infty 2m \cos \gamma t \left\{\int_0^{2\pi}\int_0^\infty \eta_o(r_o) J_o(m\bar{r}) r_o\, dr_o\, d\theta_o\right\} dm \tag{27}$$

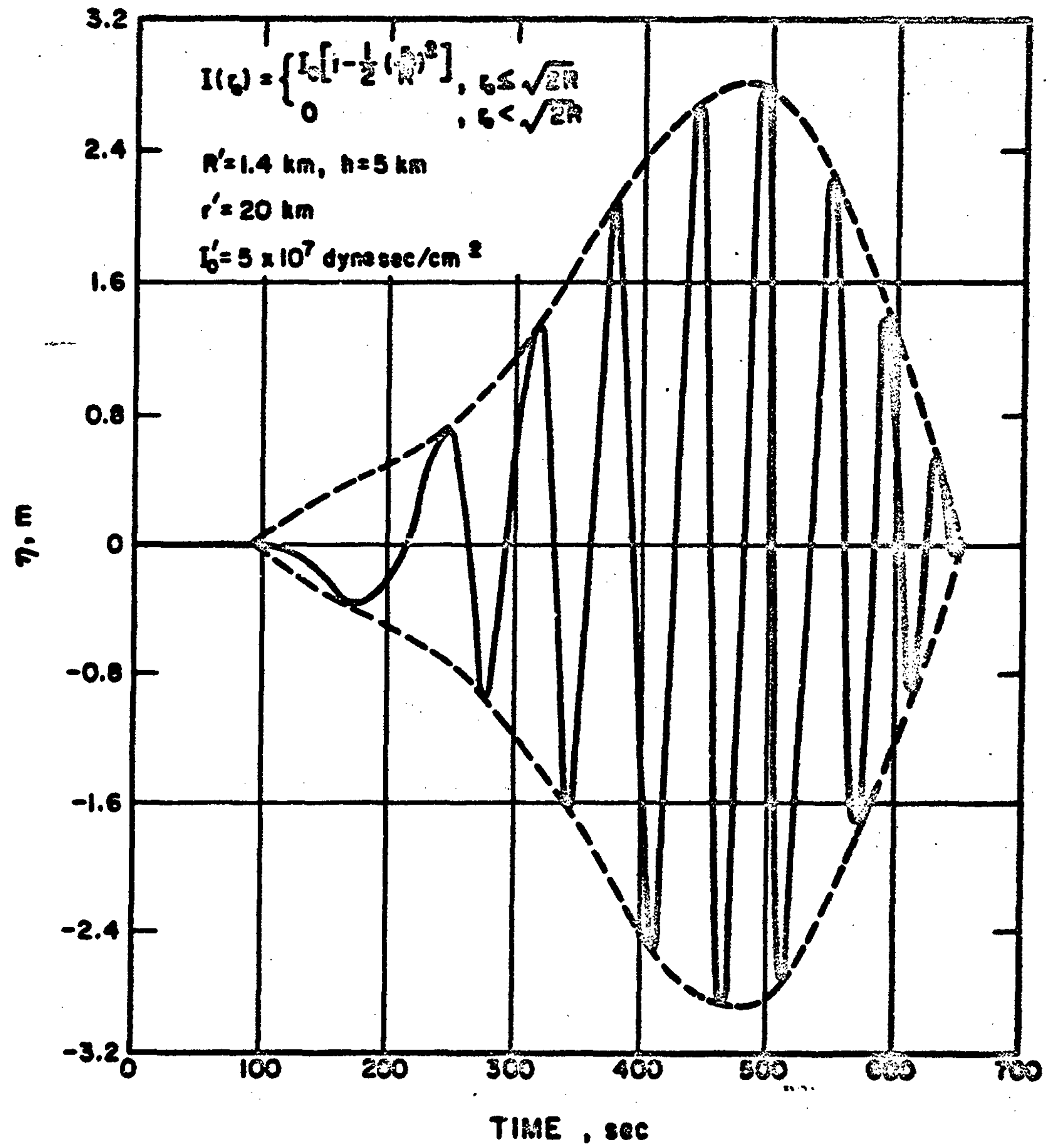

Figure 1

Wave Amplitude From an Initial Impulse

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or

$$= \frac{-1}{4\pi}\int_0^\infty 2m\cos\gamma t\left[\int_0^{2\pi}\int_0^\infty \eta_o(r_o)\left\{J_o(mr)J_o(mr_o)+2\sum_{n=1}^{\infty}J_n(mr)J_n(mr_o)\cos n(\theta-\theta_o)\right\} r_o\,dr_o\,d\theta_o\right] dm \tag{28}$$

however

$$\int_0^{2\pi} \cos n\,(\theta - \theta_o)\, d\,\theta_o = 0 \tag{29}$$

therefore

$$\eta(r,t) = -\int_0^\infty m\, J_o(mr)\cos\gamma t\left[\int_0^\infty \eta_o(r_o)\, J_o(mr_o)\, r_o\, dr_o\right] dm \tag{30}$$

where

$$\int_0^\infty \eta_o(r_o)\, J_o(mr_o)\, r_o\, dr_o = \overline{\eta}_o(m) \tag{31}$$

which is the zero order Hankel Transform of $\eta_o(r_o)$. The wave amplitude becomes

$$\eta(r,t) = -\int_0^\infty \overline{\eta}_o(m)\, m\cos\left(\sqrt{m\tanh m}\; t\right) J_o(mr)\, dm \tag{32}$$

The above integral is evaluated by the method of stationary phase where it is assumed that $\overline{\eta}_o(m)$ has a much lower frequency of oscillation than $J_o(mr)$ and the circular function. This assumption is a valid one for the types of initial deformations which are representative of underwater explosions, as can be observed from the examples in Table I. $J_o(mr)$ is approximated by an asymptotic expansion for large argument and the integral becomes

$$\eta(r,t) = \frac{1}{r}\sqrt{\frac{m\,[\text{illegible}](m)}{-\varphi'(m)}}\;\overline{\eta}_o(m)\cos\left(mr - \sqrt{m\tanh m}\; t\right) \tag{33}$$

where

$$\varphi(m) = \frac{1}{2}\sqrt{\frac{\tanh m}{m}} + \frac{1}{2\cosh^2 m}\sqrt{\frac{m}{\tanh m}} = \frac{r}{t} \tag{34}$$

The following table gives $\bar{\eta}_o(m)$ for various initial deformations $\eta_o(r_o)$.

The wave amplitude may be computed either as a function of time with r constant or as a function of r with time constant.

Figure 2 shows the wave amplitude as a function of time for example 3 of Table I at one observation station.

3.3 Axially Symmetric Time-Dependent Surface Deformation

There has been little investigation into the effect of using a time-dependent symmetric surface deformation. This section presents the theory with several specific examples given in detail. The examples can be compared with those where the source is independent of time. The intial conditions become:

a) The water surface is initially undisturbed and is defined by

$$\eta_o(r_o, \tau) = \begin{cases} \eta_o(r_o) \sin \frac{\pi}{2} \frac{\tau}{\tau^*}, & 0 \le \tau \le \tau^* \\ 0, & \tau > \tau^* \end{cases}$$

It is assumed that the variables r_o and τ are separable and τ^* should be taken as the dimensionless period of the first expansion of the bubble or the water cavity for the case of a surface explosion.

b) The initial velocity of the surface at time $\tau = 0$ is

$$\varphi_{z_o}\Big|_{\substack{z_o = 0 \\ z = 0 \\ \tau = t = 0}} = \frac{d\eta_o(r_o, \tau)}{d\tau}\Big|_{\tau=0} = + \frac{\pi}{2\tau^*}\eta_o(r_o)$$

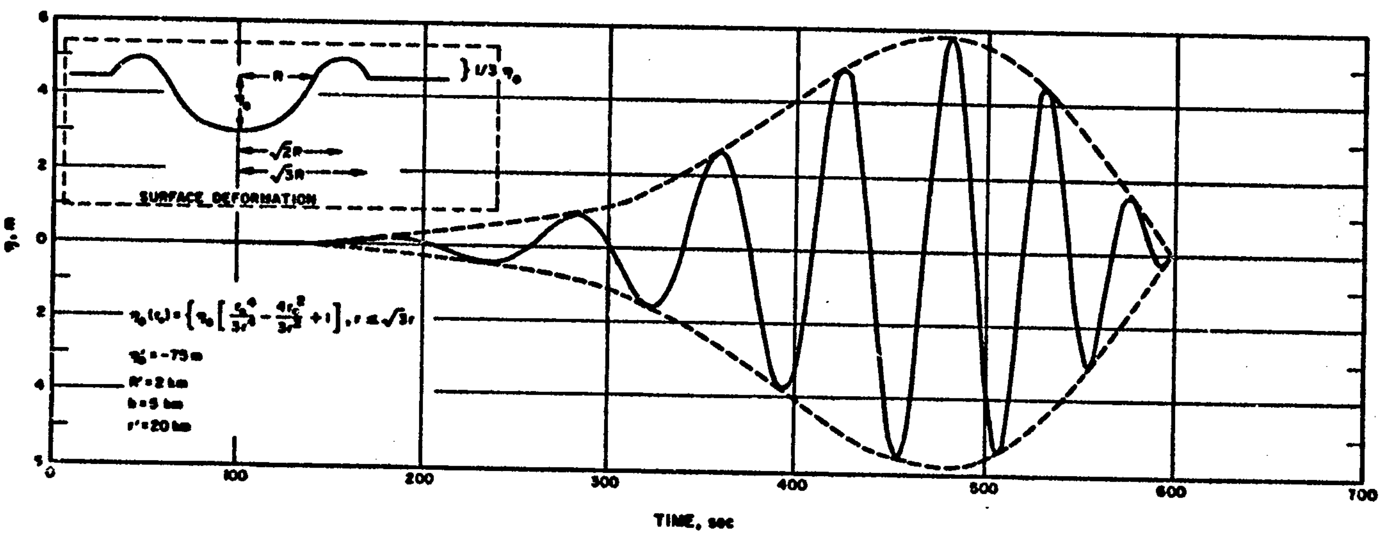

Figure 2
Wave Amplitude From a Symmetric Surface Deformation

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TABLE I
HANKEL TRANSFORM OF VARIOUS INITIAL DEFORMATIONS

	$\frac{\eta_o(r_o)}{\eta_o}$ or $\frac{I_o(r_o)}{I_o}$	$\frac{\bar{\eta}_o(m)}{\eta_o}$ or $\frac{\bar{I}_o(m)}{I_o}$
1.	$1, \; r_o \le R$ $0, \; r_o > R$	$\frac{R}{m} J_1(mR)$
2.	$\left[1 - \frac{1}{2}\left(\frac{r_o}{R}\right)^2\right], \; r_o \le \sqrt{2}R$ $0 \quad , \; r_o > \sqrt{2}R$	$\frac{2}{m^2} J_2(\sqrt{2}mR)$
3.	$\left[\frac{r_o^4}{3R^4} - \frac{4r_o^2}{3R^2} + 1\right], \; r_o \le \sqrt{3}R$ $0 \quad , \; r_o > \sqrt{3}R$	$\frac{4}{m^2} J_4(\sqrt{3}mR)$
4.	$\left[1 + 2\left(\frac{r}{R}\right)^2\right]^{-3/2}$	$\frac{R^2}{2} e^{-Rm\sqrt{2}}$
5.	$e^{-\sqrt{2}\, r/R}$	$\frac{R^2}{2(1 + R^2m^2/2)^{3/2}}$
6.	$e^{-(r/R)^2}$	$\left(\frac{R^2}{r}\right) e^{-m^2R^2/4}$

c) $\quad I_1 = \left(G_t \varphi_{z_o}\right)\Big|_{\tau=0,\, z_o=0,\, z=0}$ and

$$G_t = \int_0^\infty \frac{2}{\gamma} \sin \gamma t \; m J_o(m\bar{r})\, dm$$

d) $\quad I_2 = I_3 = 0$

Upon substitution in Eq. 16 and expressing the wave amplitude in cylindrical coordinates

$$\eta(r,t) = \frac{1}{4\pi}\int_0^\infty \frac{2m \sin\gamma t}{\gamma}\left[\int_0^{2\pi}\int_0^\infty J_o(m\bar{r})\,\eta_o(r_o)\left(\frac{+\pi}{2\tau^*}\right) r_o\, dr_o\, d\theta_o\right] dm$$

$$= \frac{+1}{4\tau^*}\int_0^\infty \frac{m \sin\gamma t}{\gamma}\left[\int_0^{2\pi}\int_0^\infty \left\{J_o(mr)J_o(mr_o) + 2\sum_{n=1}^{\infty} J_n(mr)J_n(mr_o)\cos n(\theta-\theta_o)\right\}\right.$$

$$\left. \eta_o(r_o) r_o dr_o d\theta_o\right] dm$$

$$= +\frac{2\pi}{4\tau^*}\int_0^\infty \frac{m \sin\gamma t}{\gamma}\; J_o(mr)\left[\int_0^\infty \eta_o(r_o)\; J_o(mr_o)\; r_o\; dr_o\right] dm \qquad (35)$$

However, the integral with respect to r_c is merely the zero order Hankel Transform of the initial deformation $\eta_o(r_o)$ and will be denoted by $\overline{\eta_o}(m)$.

$$\eta(r,t) = \frac{+\pi}{2\tau^*}\int_0^\infty \frac{m J_o(mr)}{\sqrt{m \tanh n}}\;\overline{\eta_o}(m)\, \sin\left(\sqrt{m \tanh m}\; t\right) dm \qquad (36)$$

Upon evaluation by the method of stationary phase

$$\eta(r,t) = \frac{+\pi}{2\tau^*}\,\frac{\overline{\eta_o}(m)}{r\sqrt{\tanh m}}\,\sqrt{\frac{\varphi(m)}{-\varphi'(m)}}\;\sin\left(mr - \sqrt{m \tanh m}\;\; t\right) \qquad (37)$$

where φ and φ' are defined as before. Examples of $\overline{\eta_o}(m)$ are found in Table I.

Figure 3 shows the wave amplitude as a function of time at one observation station where $\overline{\eta_o}(m)$ is given by example 3 of Table I.

3.4 Asymmetric Time-Dependent Surface Deformation

In addition to assuming the source of disturbance to be time-dependent, it is relatively straightforward to avoid the assumption of axial symmetry. However, in the case of explosion waves, this assumption is not necessary unless an unsymmetric cavity is produced by an incomplete detonation or by multiple detonations. The initial condition becomes:

a) The water surface is initially undisturbed and is defined by

$$\eta_o(r_o, \theta_o, t) = \begin{cases} \eta_o(r_o)\theta(\theta_o)\, T(\tau), & 0 \le \tau \le \tau^*, \quad r_o \le R \\ & 0 \le \theta_o < 2\pi \\ 0 & t > \tau,^o \quad r_o > R \end{cases}$$

It is assumed that the variables are separable and that τ^* is a parameter which should be related to the time of the initial expansion of the bubble.

b) The initial velocity of the surface at time $\tau = 0$ is

$$\varphi_{z_o}\Big|_{\substack{z_o = 0 \\ z = 0 \\ \tau = 0}} = \frac{d\,\eta_o(r_o, \theta_o, \tau)}{d\tau}\Big|_{\tau=0} = \eta_o(r_o)\, \theta(\theta_o)\, T_\tau(0)$$

c) $$I_1 = G_t\, \varphi_{z_o}\Big|_{\tau=0,\, z_o=0,\, z=0}$$ and

$$G_t = \int_0^\infty \frac{2}{\gamma} \sin \gamma t\, m\, J_o(m\bar{r})\, dm$$

d) $$I_2 = I_3 = 0$$

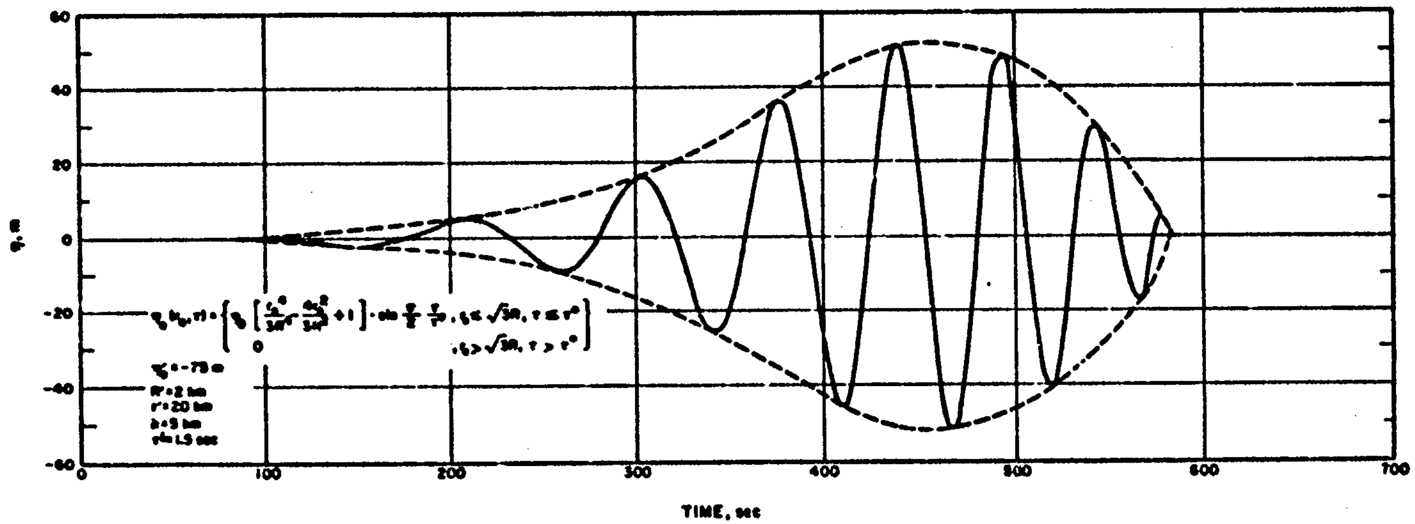

Figure 3

Wave Amplitude From a Time Dependent Symmetric Surface Deformation

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Upon substitution in Eq. 16 and expressing the wave amplitude in cylindrical coordinates

$$\eta(r,\theta,t) = \frac{T_\tau(0)}{2\pi}\int_0^\infty \frac{m\sin\gamma t}{\gamma}\left[\int_0^{2\pi}\int_0^\infty\left\{J_o(mr)J_o(mr_o)+2\sum_{n=1}^{\infty}J_n(mr)J_n(mr_o)\cos n(\theta-\theta_o)\right\}\cdot\eta_o(r_o)\,\theta(\theta_o)\,r_o\,dr_o\,d\theta_o\right]dm \quad (33)$$

Upon performing the integrations with respect to r_o and θ_o

$$\eta(r,\theta,t) = \frac{T_\tau(0)}{2\pi}\int_0^\infty \frac{m\sin\gamma t}{\gamma}\left[J_o(mr)\,\overline{\eta_o}(m)\,I_o + 2\sum_{n=1}^{\infty}J_n(mr)\,\overline{\eta_n}(m)\,I_n(\theta)\right]dm \quad (39)$$

where

$\overline{\eta_o}(m) = \int_0^\infty \eta_o(r_o)\,J_o(mr_o)\,r_o\,dr_o$ = zero order Hankel transform of $\eta_o(r_o)$

$\overline{\eta_n}(m) = \int_0^\infty \eta_o(r_o)\,J_n(mr_o)\,r_o\,dr_o$ = n^{th} order Hankel transform of $\eta_o(r_o)$

$$I_o = \int_0^{2\pi}\theta(\theta_o)\,d\theta_o \quad (40)$$

$$I_n(\theta) = \begin{cases} A_n\pi\sin n\theta \\ B_n\pi\cos n\theta \end{cases} \quad (41)$$

and the A_n are coefficients of the Fourier sine series representing $\theta(\theta_o)$, i.e., if $\theta(\theta_o) = \sum_{p=0}^{\infty} A_p \sin p\theta_o$ or if $\theta(\theta_o) = \sum_{p=0}^{\infty} B_p\cos p\theta_o$, then the B_n are the coefficients of the Fourier cosine series and $p = n$.

The above integral is evaluated by the method of stationary phase and

$$\eta(r,\theta,t) = \frac{T_\tau(0)}{2\pi r\sqrt{\tanh m}}\sqrt{\frac{\varphi(m)}{-\varphi'(m)}}\left\{I_o\,\overline{\eta_o}(m) + 2\sum_{n=1}^{\infty}\overline{\eta_n}(m)\,I_n(\theta)\right\}\sin\left(mr - \sqrt{m\tanh m}\;t\right) \quad (42)$$

The following table illustrates the various parameters in the above solution for specific initial deformations.

	$\eta_o(r_o)$	$\theta(\theta_o)$	$T(\tau)$	I_o	$I_n(0)$	$T_\tau(0)$
1.	$\begin{cases} \eta_o, & r_o \le R \\ 0, & r_o > R \end{cases}$	$\frac{2N - 1 + \cos\theta_o}{2N}$, $N \ge 1$	$\sin\frac{\pi}{2}\frac{\tau}{\tau^*}$, $0 \le \tau < \tau^*$	$\frac{\pi(2N-1)}{N}$	$\frac{\cos\theta}{2N}$	$\frac{\pi}{2\tau^*}$
2.	$\begin{cases} \eta_o\left[1 - \frac{1}{2}\left(\frac{r_o}{R}\right)^2\right], & r_o \le \sqrt{2}\,R \\ 0, & r_o > \sqrt{2}\,R \end{cases}$	$\frac{2N - 1 + \cos\theta_o}{2N}$, $N \ge 1$	$\sin\frac{\pi}{2}\frac{\tau}{\tau^*}$, $0 \le \tau < \tau^*$	$\frac{\pi(2N-1)}{N}$	$\frac{\cos\theta}{2N}$	$\frac{\pi}{2\tau^*}$
3.	$\begin{cases} \eta_o\left[\frac{r_o^4}{3R^4} - \frac{4r_o^2}{3R^2} + 1\right], & r_o \le \sqrt{3}\,R \\ 0, & r_o > \sqrt{3}\,R \end{cases}$	$\frac{2N - 1 + \cos\theta_o}{2N}$, $N \ge 1$	$\sin\frac{\pi}{2}\frac{\tau}{\tau^*}$; $0 \le \tau < \tau^*$	$\frac{\pi(2N-1)}{N}$	$\frac{\cos\theta}{2N}$	$\frac{\pi}{2\tau^*}$

$\overline{\eta_n}(m)$

1. $$\begin{cases} \overline{\eta_1}(m) = \dfrac{\eta_o R}{m} \displaystyle\sum_{k=0}^{\infty} \frac{(k+1)}{(k+3/2)(k+1/2)}\, J_{2(k+1)}(mR) \\ \overline{\eta_n}(m) = 0, \quad n \ge 2 \end{cases}$$

2. $$\begin{cases} \overline{\eta_1}(m) = \dfrac{\sqrt{2}\,\eta_o R}{m} \displaystyle\sum_{k=0}^{\infty} \frac{(k+1)}{(k+3/2)(k+1/2)} \left\{1 + \frac{3}{4(k+3/2)(k-1/2)}\right\} J_{2(k+1)}(\sqrt{2}\,mR) \\ \overline{\eta_n}(m) = 0, \quad n \ge 2 \end{cases}$$

3. $$\begin{cases} \overline{\eta_1}(m) = \dfrac{\sqrt{3}\,\eta_o R}{m} \displaystyle\sum_{k=0}^{\infty} \frac{(k+1)}{(k+3/2)(k+1/2)} \left\{1 + \frac{3}{(k+3/2)(k-1/2)} + \frac{135}{(k+\frac{7}{2})(k+\frac{5}{2})(k-\frac{1}{2})(k-\frac{3}{2})}\right\} J_{2(k+1)}(\sqrt{3}\,mR) \\ \overline{\eta_n}(m) = 0, \quad n \ge 2 \end{cases}$$

Figures 4, 5, 6, 7, 8, 9, 10, 11, and 12 show the wave amplitude as a function of time at six observation stations which are equidistant, r, from the origin and at various locations θ on the surface where the initial conditions are taken from example 3 of Table II. A comparison with Fig. 3 of the previous section shows the effect of considering an asymmetric time dependent source where the only difference is the function $\theta(\theta_0)$. It should be noted that the consideration of an asymmetric source is of little value when considering the effects of a single detonation unless it happens to be an incomplete and asymmetrical detonation. However, the method is certainly applicable to multiple detonations.

The physical significance of considering the initial deformation of this form should be clearly understood. It appears, at a cursory glance, that we have obtained an asymptotic solution, valid at large distances from the source, which yields the solution for a deformation that forms in a prescribed manner from time 0 to τ^*, and that the solution is valid for all $t \geq \tau^*$. However, this is not precisely the case. Actually, we are solving an initial value problem where we prescribe the shape of the water surface and the velocity field at the surface at time $t = \tau = 0$. The velocity field is obtained from the prescribed time dependent deformation from time 0 to τ^*. The result is that the solution obtained will be a solution for a class of initial conditions, precisely that class which has the same surface shape and velocity field at the surface at time $\tau = 0$. This class is described by the set S consisting of elements $T(\tau)$ such that

$$T(0) = 0 \qquad \text{and} \qquad T_\tau(0) = \frac{\pi}{2\tau^*}$$

Examples of this set include

$$\begin{aligned} T(\tau) &= \sin\frac{\pi}{2}\frac{\tau}{\tau^*} + \tau^n + \tau^n \cos a\tau, \ n \geq 2 \\ &= \sin\frac{\pi}{2}\frac{\tau}{\tau^*} + \tau^n e^{b\tau^p}, \ n \geq 2 \\ &= \frac{\pi}{2}\frac{\tau}{\tau^*} \end{aligned}$$

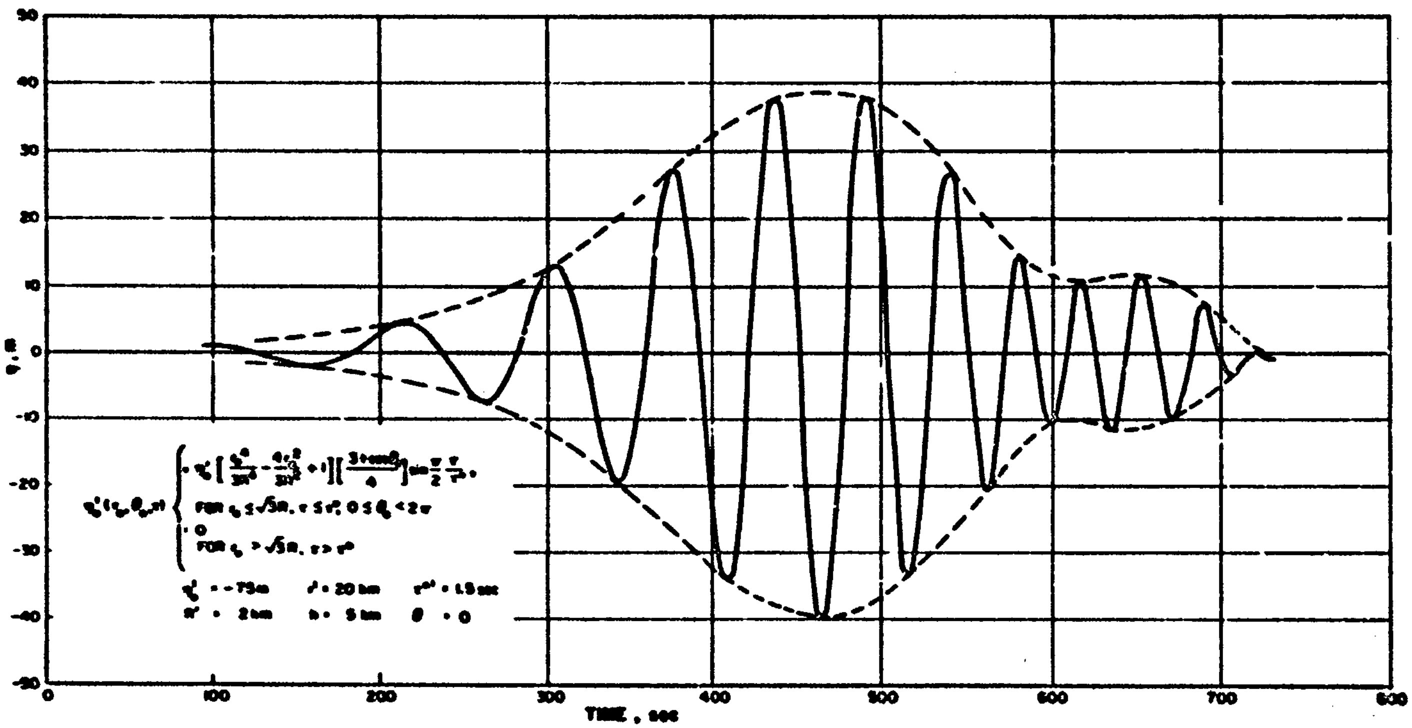


Figure 4
Wave Amplitude From an Asymmetric Time Dependent Surface Deformation

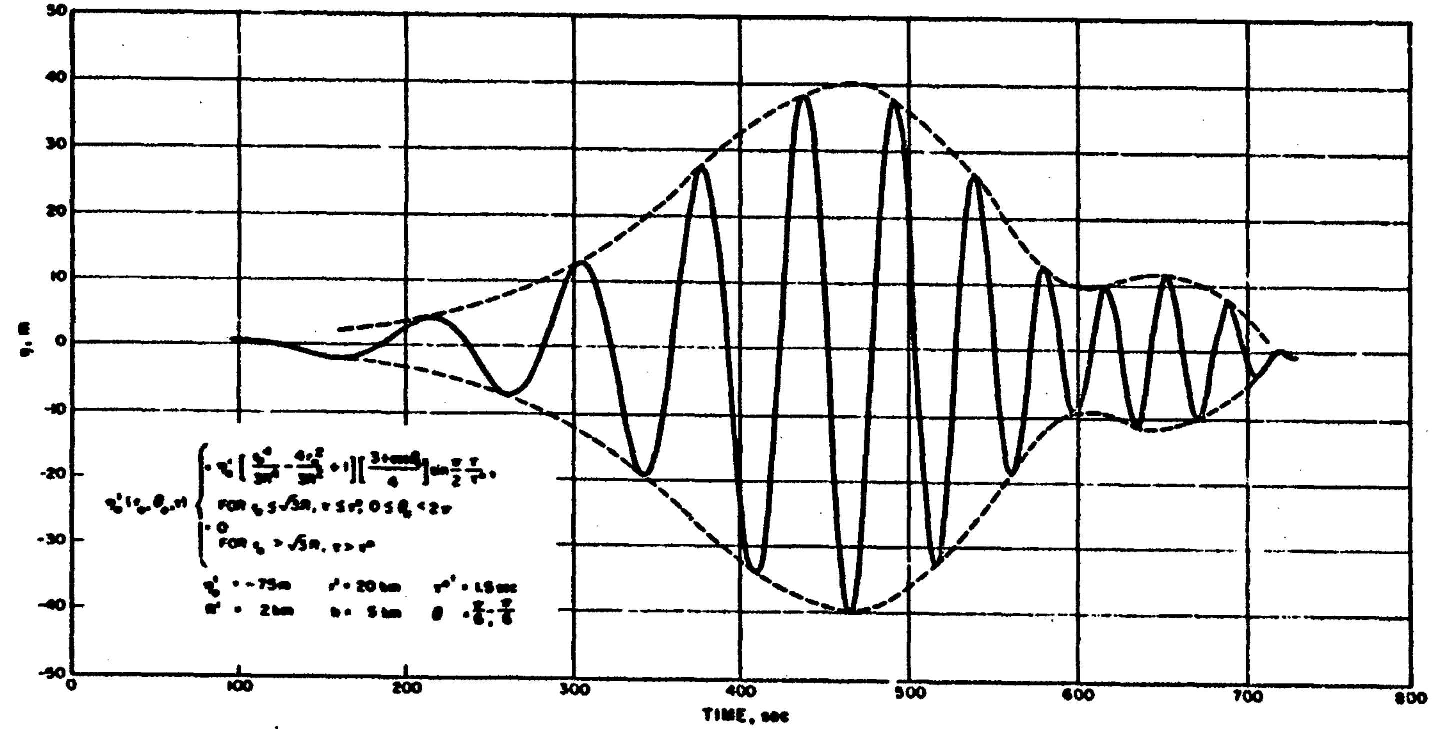


Figure 5

Wave Amplitude From an Asymmetric Time Dependent Surface Deformation

PB-3-7304

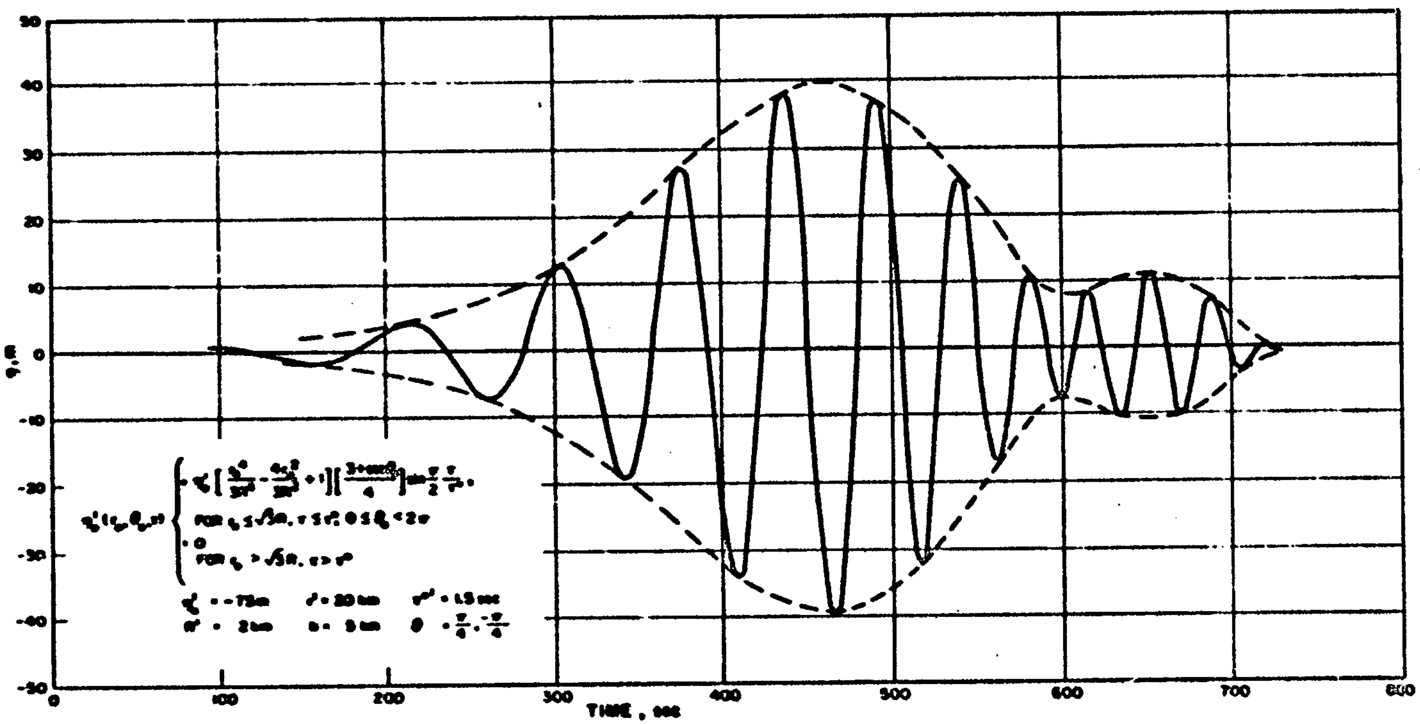


Figure 6

Wave Amplitude From an Asymmetric Time Dependent Surface Deformation

PB-3-7305

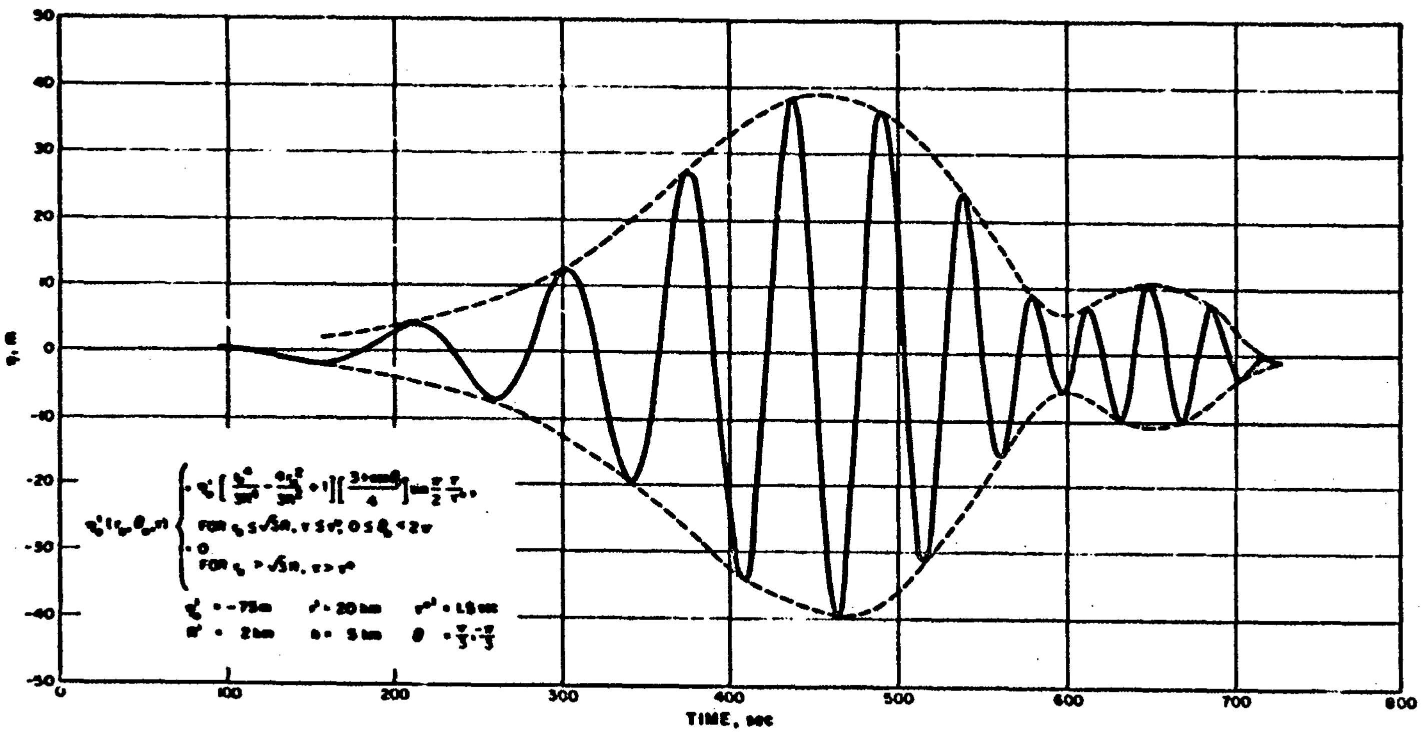


Figure 7
Wave Amplitude From an Asymmetric Time Dependent Surface Deformation

PB-3-7386

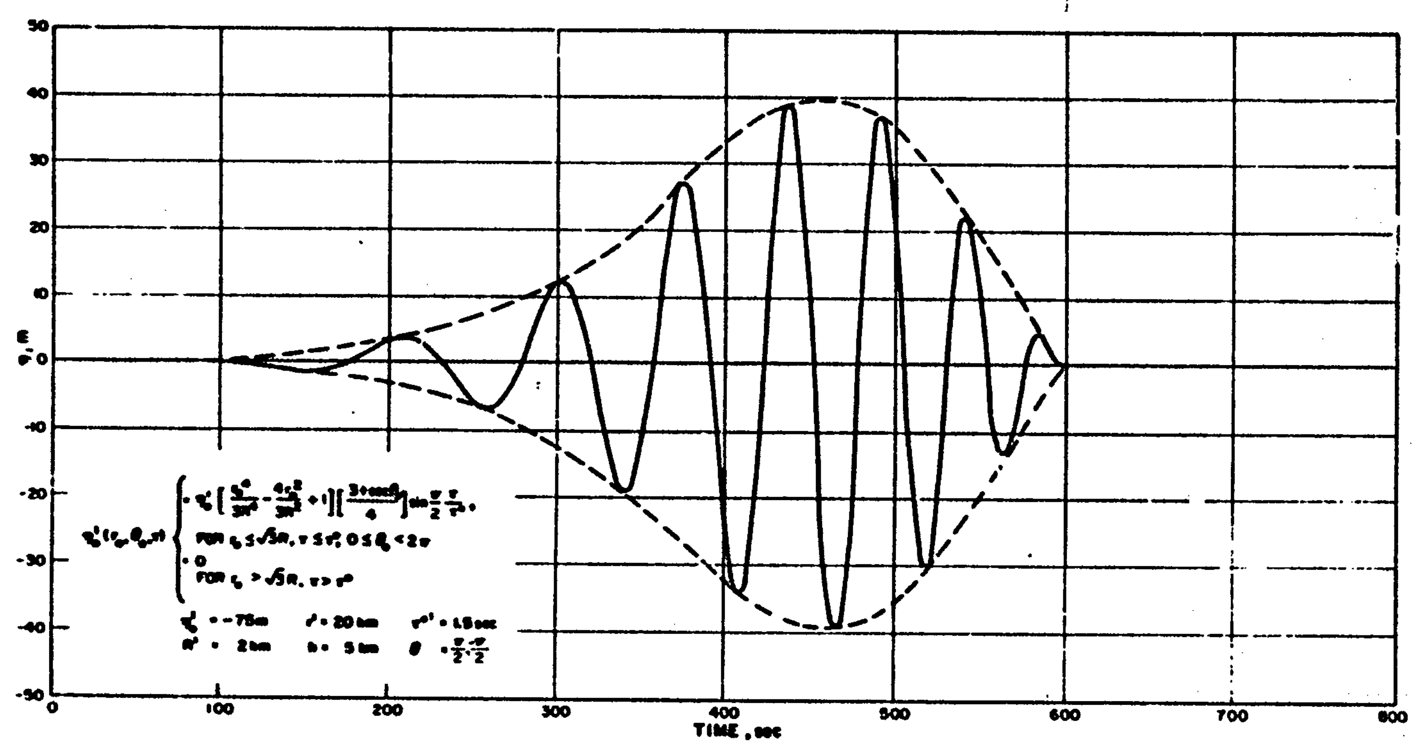


Figure 8
Wave Amplitude From an Asymmetric Time Dependent Surface Deformation

PB-3-7387

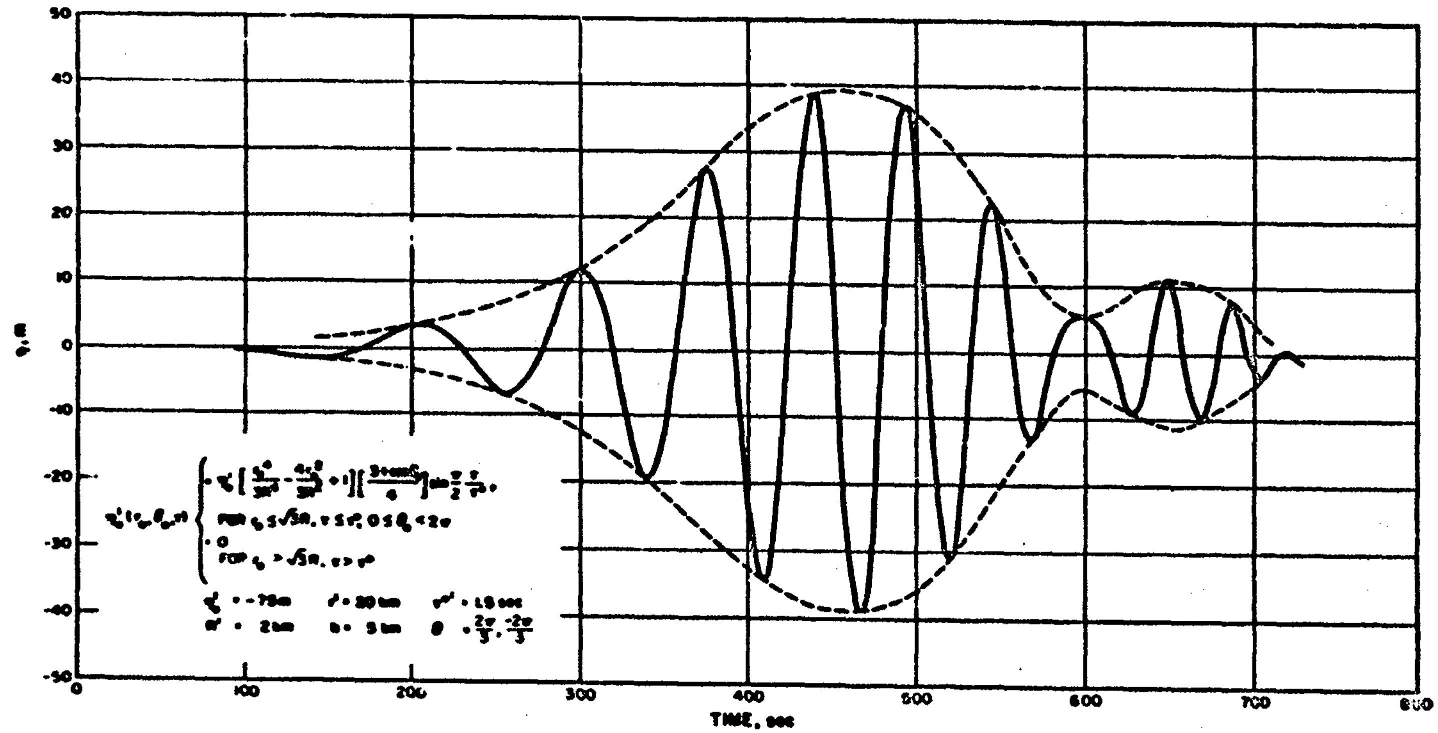


Figure 9

Wave Amplitude From an Asymmetric Time Dependent Surface Deformation

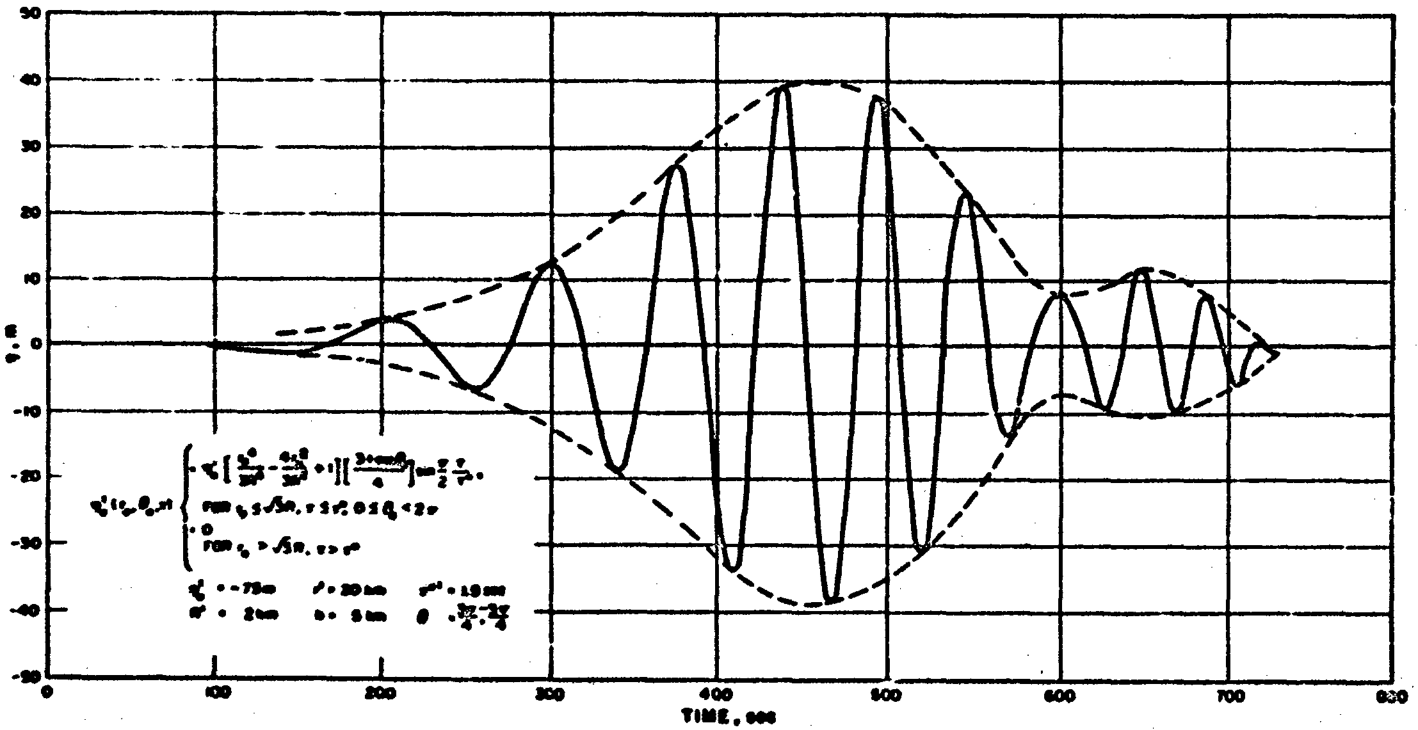


Figure 10
Wave Amplitude From an Asymmetric Time Dependent Surface Deformation

PB-3-7389

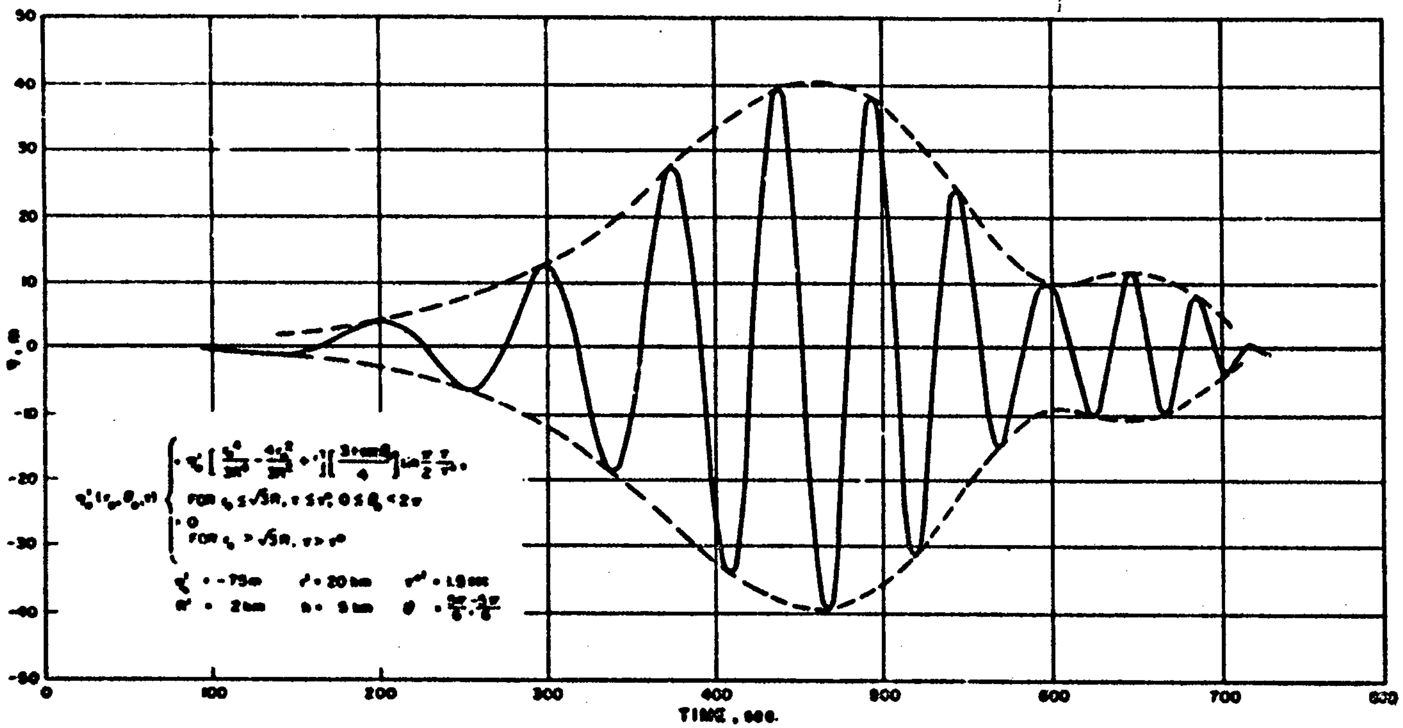


Figure 11
Wave Amplitude From an Asymmetric Time Dependent Surface Deformation

PB-3-7390

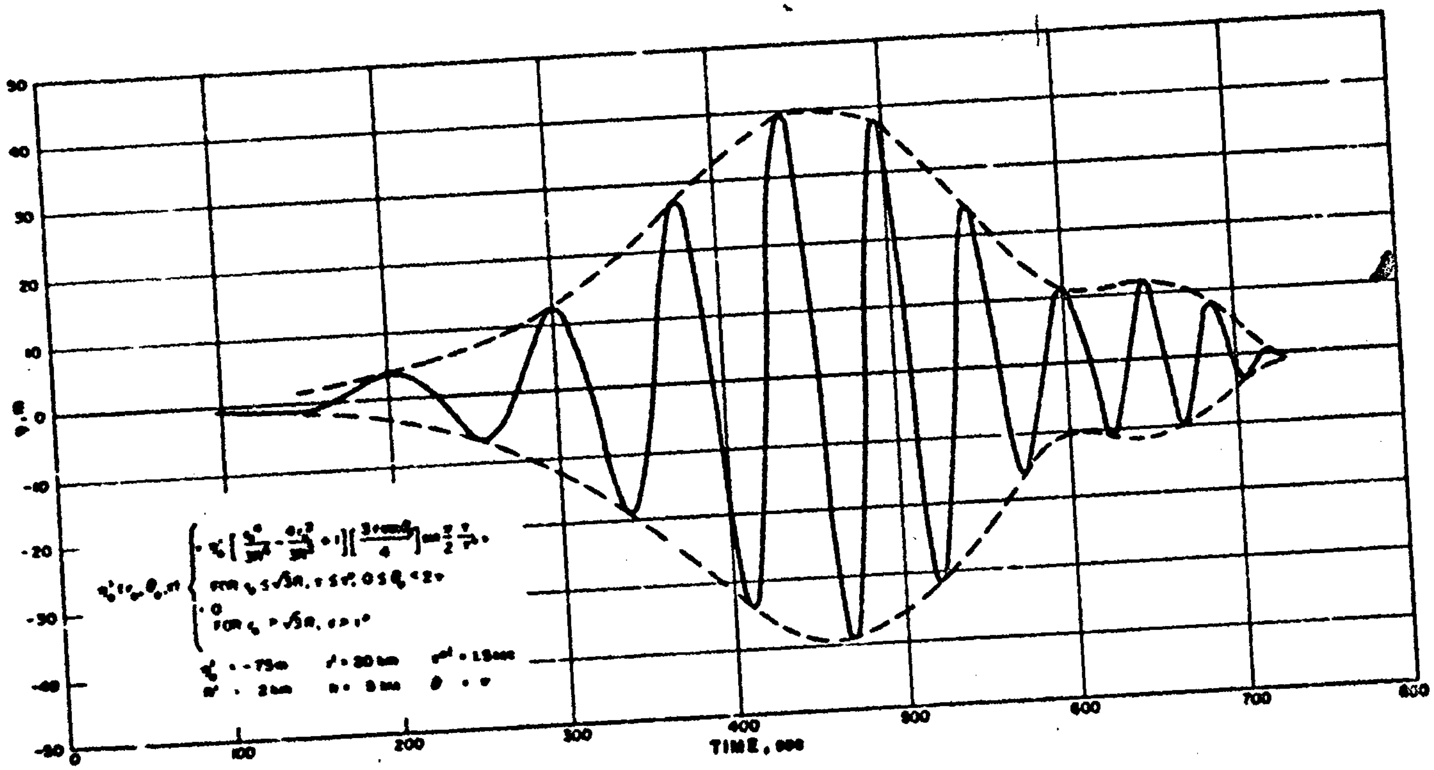


Figure 12
Wave Amplitude From an Asymmetric Time Dependent Surface Deformation

PB-3-7391

That is, the solution does not depend on the precise manner in which the deformation builds up from time 0 to τ^* but only on how it starts to build up. Furthermore, if the solution were evaluated at $t = \tau^*$, it would not yield the precise deformation assumed (or thought to be assumed). However, it can be expected that the solution probably would not deviate significantly from $\eta_o(r_o, \theta_o, \tau^*)$ since the assumed initial conditions are physically reasonable conditions and the mathematical model also is physically reasonable.

4. METHOD OF SOLUTION IN THE AREA NEAR THE SOURCE

The asymptotic solutions (method of stationary phase) presented in the previous section are not valid in the area near the explosion. The integral with respect to m must be evaluated in some other approximate manner and the method proposed by Whalin (1965) for a symmetric water cavity is also applicable with only slight modification. The equation to be solved, in general, is

$$\eta(r,\theta,t) = \frac{1}{2\pi}\int_0^\infty \frac{m \sin \gamma t}{\gamma}\left[I_o J_o(mr)\,\overline{\eta_o}(m) + 2\sum_{n=1}^{\infty} I_n(\theta) J_n(mr)\,\overline{\eta_n}(m)\right] dm$$

where the quantities I_o, $\overline{\eta_o}(m)$, $I_n(\theta)$, $\overline{\eta_n}(m)$, and γ are the same as previously defined. The proposed method of evaluation is to fit a fourth degree polynomical to each half oscillation of the integrand and integrate directly. This method requires that all computations be performed in double precision and the upper limit of integration may be chosen to bound the error to almost any predetermined limit.

The above integral is to be approximated in the following manner

$$\eta(r,\theta,t) = \frac{1}{2\pi}\int_0^a g(m)\,dm + \frac{1}{2\pi}\int_a^\beta \frac{f(m)}{m^{1/2}}\,dm + \frac{1}{2\pi}\int_\beta^\infty h(m)\,dm$$

where

a = a small number such that the oscillatory functions of the integrand may be approximated by asymptotic expansions near $m = 0$.

β = a value of m large enough so that the contribution of the latter integral is negligible and in addition is a zero of the integrand. This must be carefully chosen and depends upon $\overline{\eta_o}(m)$ and $\overline{\eta_n}(m)$.

$g(m)$ = the integrand when approximated by asymptotic expansions for small m

Having chosen the proper limits a and β, the first integral may be integrated directly and the latter integral is negligible. The second integral will be approximated by a polynomical fit, ${}_nf_i(m)$, over each half oscillation of the integrand.

$$ {}_nf(m) = \frac{m^{3/2} \sin \gamma t}{\gamma} \left\{ I_o J_o(mr) \overline{\eta_o}(m) + 2 \sum_{n=1}^{\infty} I_n(\theta)\, J_n(mr)\, \overline{\eta_n}(m) \right\} $$

where

$$ {}_of(m) = \frac{\sin \gamma t}{\gamma} \quad J_o(mr)\, \overline{\eta_o}(m), \quad n = 0 $$

$$ {}_nf(m) = 2\, I_n(\theta)\, J_n(mr)\, \overline{\eta_n}(m), \quad n \geq 1 $$

$$ {}_nf(m) \approx {}_nf_i(m) = \sum_{i=0}^{{}_nI} \left\{ {}_nA_i'\, m^4 + {}_nB_i' m^3 + {}_nC_i' m^2 + {}_nD_i' m + {}_nE_i' \right\}, $$

$$ a \leq m \leq {}_nm_i, \quad i = 1 $$

$$ {}_nm_{i-1} \leq m \leq {}_nm_i, \quad i = 2, 3, \ldots {}_nI $$

where the ${}_nm_i$ and the ordered zeros of the integrand, ${}_nf(m)$, and ${}_nI$ is the number of zeros of the integrand from a up to and including β. Upon substitution and subsequent integration, the second integral and consequently the wave amplitude becomes

$$ \eta(r,\theta,t)\Big|_a^{\beta} = \frac{4I_o'}{\rho(gh)^{\frac{1}{2}}} \sum_{n=1}^{N} \left\{ \begin{array}{l} \frac{{}_nA_1'}{9}\left(m_1^{9/2} - a^{9/2}\right) + \frac{{}_nB_1'}{7}\left({}_nm_1^{7/2} - a^{7/2}\right) + \frac{{}_nC_1'}{5}\left({}_nm_1^{5/2} - a^{5/2}\right) \\ + \frac{{}_nD_1'}{3}\left({}_nm_1^{3/2} - a^{3/2}\right) + {}_nE_1'\left({}_nm_1^{1/2} - a^{1/2}\right) \\ + \sum_{i=2}^{i={}_nI} \left[\frac{{}_nA_i'}{9}\left({}_nm_i^{9/2} - {}_nm_{i-1}^{9/2}\right) + \frac{{}_nB_i'}{7}\left({}_nm_i^{7/2} - {}_nm_{i-1}^{7/2}\right) + \frac{{}_nC_i'}{5}\left({}_nm_i^{5/2} - {}_nm_{i-1}^{5/2}\right) \right. \\ \left. + \frac{{}_nD_i'}{3}\left({}_nm_i^{3/2} - {}_nm_{i-1}^{3/2}\right) + {}_nE_i'\left({}_nm_i^{1/2} - {}_nm_{i-1}^{1/2}\right) \right] \end{array} \right\} $$

where

$$ {}_nA_i' = {}_nA_i $$

$$ {}_nB_i' = -\left({}_nm_i + {}_nm_{i-1}\right) {}_nA_i + {}_nB_i $$

$$ {}_nC_i' = {}_nm_i \, {}_nm_{i-1} \, {}_nA_i - \left({}_nm_i + {}_nm_{i-1}\right) {}_nB_i + {}_nC_i $$

$$ {}_nD_i' = {}_nm_i \, {}_nm_{i-1} \, {}_nB_i - \left({}_nm_i + {}_nm_{i-1}\right) {}_nC_i $$

$$ {}_nE_i' = {}_nm_i \, {}_nm_{i-1} \, {}_nE_i $$

$$ {}_nA_i = \frac{-64}{\left({}_nm_i - {}_nm_{i-1}\right)^4}\left[\frac{2}{3}\, {}_nf\left(\frac{{}_nm_i + 3\,{}_nm_{i-1}}{4}\right) - {}_nf\left(\frac{{}_nm_i + {}_nm_{i-1}}{2}\right) + \frac{2}{3}\, {}_nf\left(\frac{3\,{}_nm_i + {}_nm_{i-1}}{4}\right)\right] $$

$$ \begin{aligned} {}_nB_i = \frac{64}{\left({}_nm_i - {}_nm_{i-1}\right)^4}\Bigg[& \left(\frac{5\,{}_nm_i + 3\,{}_nm_{i-1}}{6}\right) {}_nf\left(\frac{{}_nm_i + 3\,{}_nm_{i-1}}{4}\right) \\ & - \left({}_nm_i + {}_nm_{i-1}\right) {}_nf\left(\frac{{}_nm_i + {}_nm_{i-1}}{2}\right) \\ & + \left(\frac{3\,{}_nm_i + 5\,{}_nm_{i-1}}{6}\right) {}_nf\left(\frac{3\,{}_nm_i + {}_nm_{i-1}}{4}\right)\Bigg] \end{aligned} $$

$$ {}_nC_i = \frac{-8}{\left({}_nm_i - {}_nm_{i-1}\right)^4}\left[\begin{aligned} & \frac{2\left({}_nm_i + {}_nm_{i-1}\right)\left(3\,{}_nm_i + {}_nm_{i-1}\right)}{3}\, {}_nf\left(\frac{{}_nm_i + 3\,{}_nm_{i-1}}{4}\right) \\ & - \frac{\left({}_nm_i + 3\,{}_nm_{i-1}\right)\left(3\,{}_nm_i + {}_nm_{i-1}\right)}{2}\, {}_nf\left(\frac{{}_nm_i + {}_nm_{i-1}}{2}\right) \\ & + \frac{2\left({}_nm_i + 3\,{}_nm_{i-1}\right)\left({}_nm_i + {}_nm_{i-1}\right)}{3}\, {}_nf\left(\frac{3\,{}_nm_i + {}_nm_{i-1}}{4}\right) \end{aligned}\right] $$

$$ {}_nA_1' = {}_nA_1 $$

$$ {}_nB_1' = {}_nB_1 - {}_nm_1\,{}_nA_1 $$

$$ {}_nC_1' = {}_nC_1 - {}_nm_1\,{}_nB_1 $$

$$ {}_nD_1' = {}_nD_1 - {}_nm_1\,{}_nC_1 $$

$$ {}_nE_1' = -\,{}_nm_1\,{}_nD_1 $$

$$ {}_nA_1 = \frac{1}{(a - {}_nb)}\left[\frac{({}_nf_5 - {}_nf_7)}{(a - {}_nc)} - \frac{({}_nf_6 - {}_nf_7)}{({}_nb - {}_nc)}\right] $$

$$ {}_nB_1 = \frac{1}{(a - {}_nb)}\left[\frac{(a + {}_nb + {}_nc)}{({}_nb - {}_nc)}\cdot({}_nf_6 - {}_nf_7) - \frac{({}_nb + {}_nc + {}_nd)}{(a - {}_nc)}({}_nf_5 - {}_nf_7)\right] $$

$$ {}_nC_1 = \frac{({}_nb\,{}_nc + {}_nb\,{}_nd + {}_nc\,{}_nd)}{(a - {}_nb)(a - {}_nc)}\,{}_nf_5 - \frac{(a\,{}_nc + a\,{}_nd + {}_nc\,{}_nd)}{(a - {}_nb)({}_nb - {}_nc)}\,{}_nf_6 + \frac{(a\,{}_nb + a\,{}_nd + {}_nb\,{}_nd)}{(a - {}_nc)({}_nb - {}_nc)}\,{}_nf_7 $$

$$ {}_nD_1 = a\left[-\frac{({}_nb - {}_n\alpha_2\,{}_nd)}{{}_nb\,({}_nb - {}_nd)} + \frac{({}_nc - {}_n\alpha_3\,{}_nd)}{{}_nc\,({}_nc - {}_nd)}\right] + {}_nb\left[\frac{(a - {}_n\alpha_1\,{}_nd)}{a\,(a - {}_nd)} - \frac{({}_nc - {}_n\alpha_3\,{}_nd)}{{}_nc\,({}_nc - {}_nd)}\right] $$

$$ + {}_nc\left[\frac{({}_nb - {}_n\alpha_2\,{}_nd)}{{}_nb\,({}_nb - {}_nd)} - \frac{(a - {}_n\alpha_1\,{}_nd)}{a\,(a - {}_nd)}\right] $$

$$ {}_nb = {}_nm_1 + 3a $$

$$ {}_nc = {}_nm_1 + a $$

$$ {}_nd = \frac{3\,{}_nm_1 + a}{4} $$

$$ {}_nf_1 = \frac{{}_nf(a)}{{}_nm - a} $$

$$ {}_nf_2 = \frac{-4\,{}_nf\left(\frac{{}_nm_1 + 3a}{4}\right)}{3\,({}_nm_1 + a)} $$

$$ {}_nf_3 = \frac{-2\,{}_nf\left(\frac{{}_nm_1 + a}{2}\right)}{{}_nm_1 - a} $$

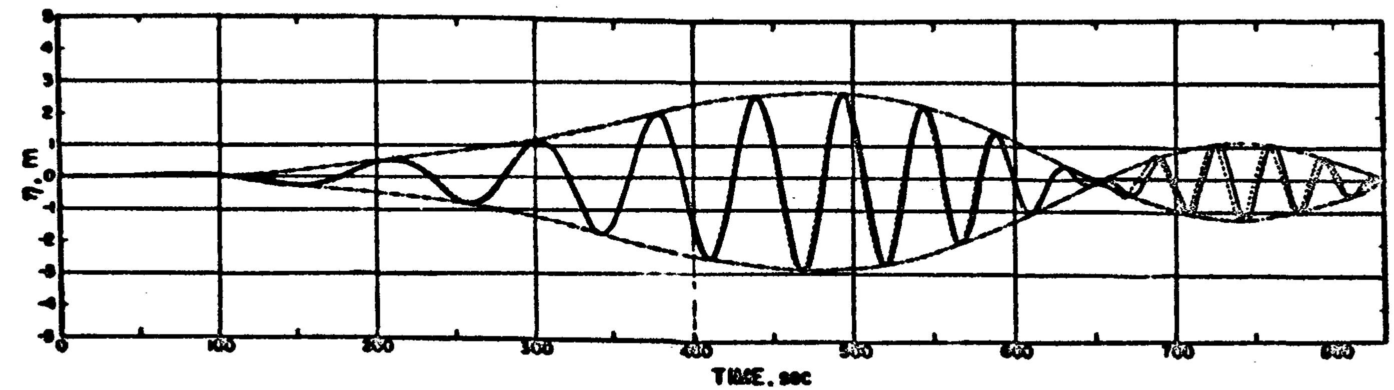
$$ {}_nf_4 = \frac{-4\,{}_nf\left(\frac{3\,{}_nm_1 + a}{4}\right)}{({}_nm_1 - a)} $$

$$ {}_nf_5 = \frac{{}_nf_1 - {}_nf_4}{a - {}_nd} \qquad {}_n\alpha_1 = {}_nf_1/{}_nf_4 $$

$$ {}_nf_6 = \frac{{}_nf_2 - {}_nf_4}{{}_nb - {}_nd} \qquad {}_n\alpha_2 = {}_nf_2/{}_nf_4 $$

$$ {}_nf_7 = \frac{{}_nf_3 - {}_nf_4}{{}_nc - {}_nd} \qquad {}_n\alpha_3 = {}_nf_3/{}_nf_4 $$

Examples of this integration method are shown in Figs. 13 to 15 for the case of a parabolic impulse and further details can be found in Whalin (1965). The method is relatively easy to use and the accuracy of the integration procedure is well established.



$$I(r_0) = \begin{cases} I_0 \left[1 - \frac{1}{2}\frac{r_0^2}{R^2}\right], & r_0 \le \sqrt{2}R \\ 0, & r_0 > \sqrt{2}R \end{cases}$$

$I_0' = 5 \times 10^7$ dyne sec/cm $r' = 20$ km

$R' = 1.4$ km $h = 5$ km

Figure 13

A Comparison of the Asymptotic Solution for a Parabolic Impulse With an Alternate Integration Method

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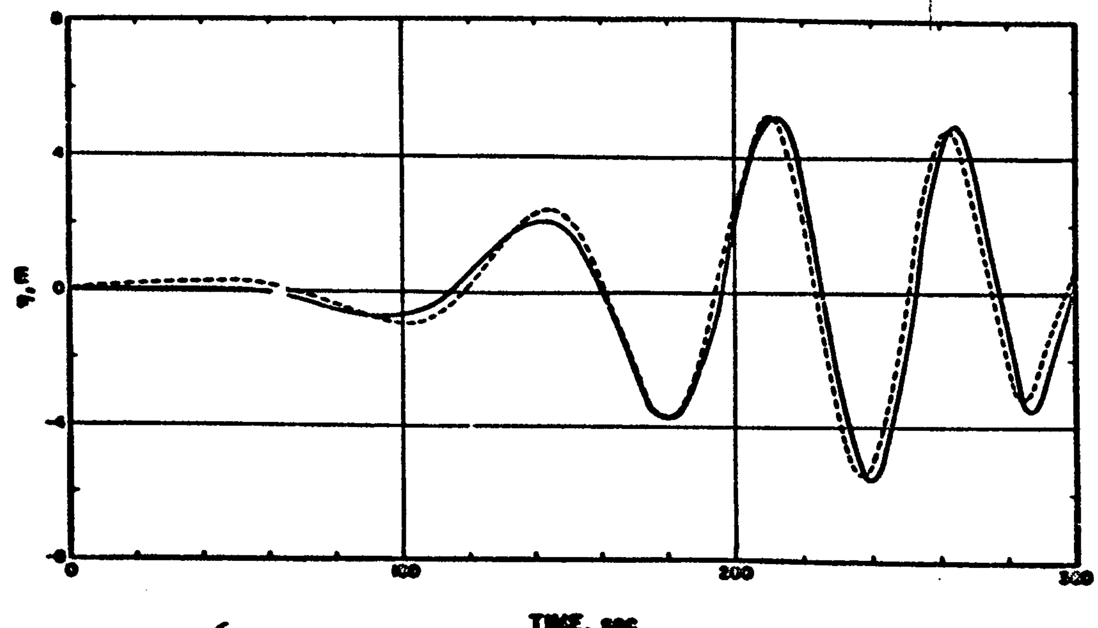


$$I(r_0) = \begin{cases} I_0 \left[1 - \frac{1}{2}\frac{r_0^2}{R^2}\right], & r_0 \le \sqrt{2}R \\ 0, & r_0 > \sqrt{2}R \end{cases}$$

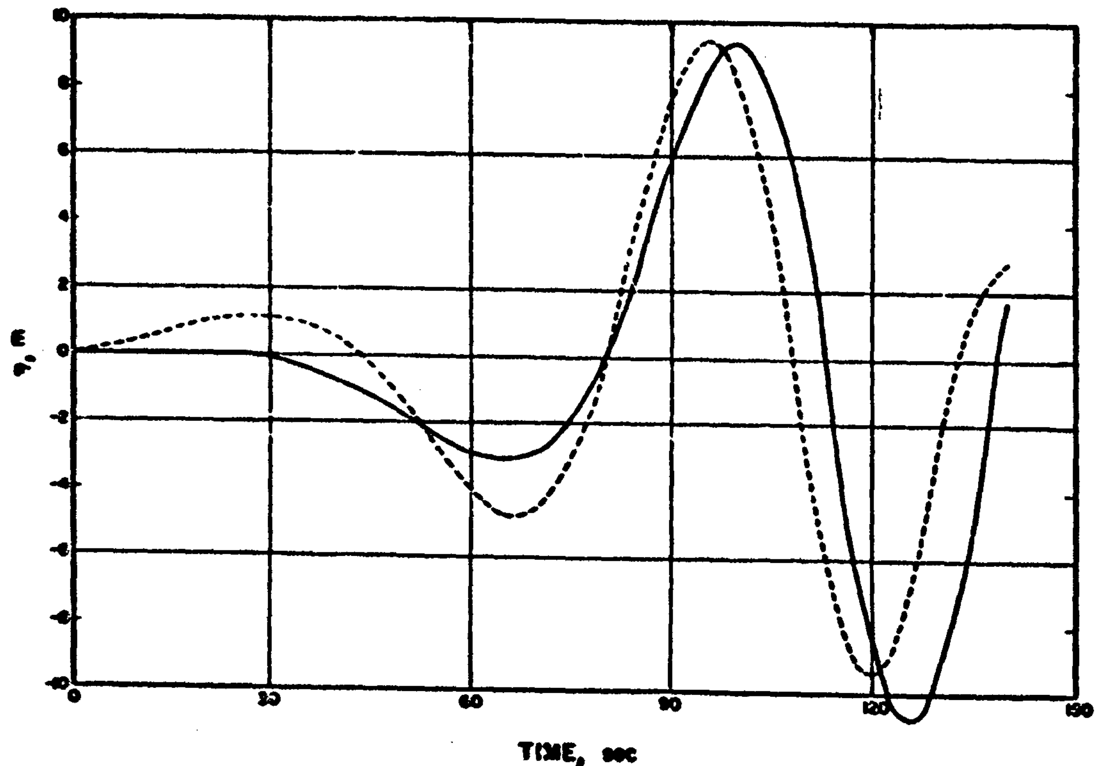
$I_0' = 5 \times 10^7$ dyne sec/cm, $r' = 10$ km

$R' = 1.4$ km, $h = 5$ km

Figure 14

A Comparison of the Asymptotic Solution for a Parabolic Impulse With an Alternate Integration Method

PA-3-7393



$$I(r_0) = \begin{cases} I_0\left[1 - \frac{1}{2}\frac{r_0^2}{R^2}\right], & r_0 \le \sqrt{2}R \\ 0, & r_0 > \sqrt{2}R \end{cases}$$

$I_0' = 5 \times 10$ dyne sec/cm, $r' = 5$ km

$R' = 1.4$ km, $h = 5$ km

Figure 15
A Comparison of the Asymptotic Solution for a Parabolic Impulse With an Alternate Integration Method

PA-3-7394

5. CONCLUSIONS

The preceeding theoretical developments present the limit to which one may employ a linear constant depth model to the problem of water waves generated by explosions. The present state of the art is at a point where much can be gained by the natural extensions of the theory presented in Sections 2, 3, and 4.

Data and methods are available which will enable one to relate the parameters of a time-dependent symmetrical cavity to the size and location of the weapon. In addition, there are sufficient data available from the Mono Lake tests, Hydra, and other WES tests to analyze the usefulness of considering cavity parameters which result from this model. Figures 2 and 3 indicate the difference obtained in the wave train by considering the time dependence. This order of magnitude difference in the envelope amplitude is a result of the fact that a symmetric stationary cavity contains only potential energy and the assumed time-dependent cavity from time 0 to τ^* will contain approximately the same potential energy at time τ^*. However, in addition, the fluid contains kinetic energy at this time and hence the resulting vast difference in the envelope amplitudes. A thorough investigation of this form of initial conditions should prove beneficial. Also, it has been pointed out by Van Dorn that one could probably even consider the effect of the plume collapse by the proper choice of initial conditions and skillful use of land cratering data.

The utility of considering an asymmetric time-dependent form as an initial condition is obvious. If the effects of multiple explosions are to be studied, then this is the only valid method. One can not, in general, merely add the solutions of several symmetric surface deformations after obtaining the asymptotic solution for each. The total deformation of the source area must be considered initially, then transformed and integrated approximately by the method of stationary phase (m. s. p) to

obtain the correct asymptotic solution. It also should be noted that the asymmetric model is applicable to the predictions of the wave spectrum generated by an earthquake when applied to circumstances consistent with the assumptions of the mathematical model.

Predictions at Mono Lake were made from example 3 of Table I which is a symmetric surface deformation surrounded by a lip such that the volume contained in the lip is equivalent to the volume of the cavity, i.e., no net addition or removal of fluid from the system.

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NOTATION

a	an integration limit
${}_nb_i$, ${}_nc_i$, ${}_nd_i$	functions of ${}_nm_i$
${}_nf_1$, ${}_nf_2$, ${}_nf_3$, ${}_nf_4$, ${}_nf_5$, ${}_nf_6$, ${}_nf_7$	functions of ${}_nm_i$
d	water depth
$g(m)$, $f(m)$ $h(m)$	functions
g	acceleration of gravity
i, j, k, n	summation indices
m	dimensionless wave number
${}_nm_i$	zeros of the integrand (i.e., the ${}_nm_i$ are the ordered zeros of ${}_nf(m)$)
p	dimensionless pressure
p'	pressure
r	dimensionless radial distance from the origin
r'	radial distance from the origin
$\bar{r}$	distance from a point (r, θ) to a source point (r_o, θ_o)
r_o, θ_o, z_o	space variables of a source point (cylindrical coordinate)
t	dimensionless time
t'	time
x, y, z	dimensionless space variables (Cartesian coordinate system)

x', y', z'	space variables (Cartesian coordinate system)
w_B	dimensionless bottom velocity
$A, {}_nA_1, {}_nA_i, {}_nA_1', {}_nA_i'$	constants
$B, {}_nB_1, {}_nB_i, {}_nB_1', {}_nB_i'$	constants
$C, {}_nC_1, {}_nC_i, {}_nC_1', {}_nC_i'$	constants
$D, {}_nD_1, {}_nD_i, {}_nD_1', {}_nD_i'$	constants
$E, {}_nE_1, {}_nE_i, {}_nE_1', {}_nE_i'$	constants
G	time-dependent Green's Function
I_o	a function describing an initial impulse
I_1, I_2, I_3	integrals
I_n	the number of zeros of the integran
$\overline{I_o}$	zero order Hankel transform of I_o
$J_n(mr)$	n^{th} order Bessel function
R	distance between points (x, y, z) and $x_o, y_o, z_o)$; also a parameter of the initial disturbance
S	a set; also a surface
$T(\tau)$	a function describing the time dependence of the initial deformation

V	dimensionless velocity
V'	velocity
${}_n\alpha_1, {}_n\alpha_2, {}_n\alpha_3$	functions of ${}_n m_i$
β	an integration limit
γ	$\sqrt{m \tanh m}$
η	dimensionless wave amplitude
η'	wave amplitude
$\overline{\eta}_n$	n^{th} order Hankel transform of η_o
η_o	function describing the initial deformation
λ	dimensionless wave length
λ'	wave length
π	constant
ρ	water density
τ	a dimensionless time variable of the source
τ^*	a constant
τ'	a time variable of the source
ϕ	velocity potential (subscripts denote partial differentiation)

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Gravity waves generated by underwater explosions and the wave run-up are predicted for a series of tests to be performed in Mono Lake (California). The time histories of the wave profiles are given at various locations in the lake in deep and shallow water where wave recorders will be installed. The times of arrival and heights of wave run-up are determined at three locations on the shoreline of Mono Lake.

This prediction is done by making use of the most advanced theories and all available information in that field, and some new theoretical developments are presented in Volumes II and III, namely:

1) A linear theory for waves generated by explosions, making use of a symmetric and asymmetric time dependent surface disturbance.

2) A linear theory for the propagation of periodic waves over the continental slope.

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